Lesson 28 – Higher Order Partial Derivatives

Exam 3

Posted Oct 30, 2023 10:38 AM

Exam 3 is Wednesday, November 8 from 6:30pm-7:30pm. The location of the exam will depend on your instructor (and please note that these locations are different than the first 2 exams):

Ben Doyle: LILY 1105

Victor Hughes: CL50 224

Jakayla Robbins: LILY 1105

Alexandra Cuadra: CL50 224

Dave Norris: CL50 224

There is an exam memo in Brightspace under Contents->Exam Information with more detailed information about the exam. You will be emailed a seating assignment closer to the exam date.

Attachment(s):

Exam3Memo.pdf (41.65 KB)

Quiz - Friday 11/10/23 – Lesson 28

Lesson 28 – Higher Order Partial Derivatives

I. Notation

II. Examples

I. Notation

Last class: f_x, $\frac{\partial z}{\partial x}$ – pretend y is a constant

$z = f(x, y)$ $\quad$ f_y, $\frac{\partial z}{\partial y}$ – pretend x is a constant

Today

$$f_{xx} = (f_x)_x \qquad \frac{\partial^2 z}{\partial x^2}$$

$$f_{yy} = (f_y)_y \qquad \frac{\partial^2 z}{\partial y^2}$$

$$f_{xy} = (f_x)_y \qquad \frac{\partial^2 z}{\partial y \partial x}$$

1. Take partial with respect to x
2. Then take partial of the result with respect to y.

Similar. $f_{yx} = (f_y)_x$

$\boxed{Ex}$ $f(x,y) = x^4 y^2 - x^3 y$

Last class, but you need these to find f_{xx}, f_{yy}, f_{xy}, and f_{yx}

$$\begin{cases} f_x(x,y) = 4x^3y^2 - 3x^2 y \\ f_y(x,y) = x^4 \cdot 2y - x^3 \cdot 1 \quad = 2x^4 y - x^3 \end{cases}$$

$f_{xx}(x,y) = f_x\left[4x^3y^2 - 3x^2 y\right] = 12x^2y^2 - 6xy$

$f_{xy}(x,y) = f_y\left[4x^3y^2 - 3x^2y\right] = 4x^3 \cdot 2y - 3x^2 \cdot 1$
$= 8x^3y - 3x^2$

$f_{yy}(x,y) = f_y\left[2x^4 y - x^3\right] = 2x^4 \cdot 1 - 0$
$= 2x^4$

1st

$f_{yx}(x,y) = f_x\left[2x^4y - x^3\right] = 8x^3y - 3x^2$

1st

$\boxed{Ex}$ $f(x,y) = xe^{-3y} + \sin(2x-5y)$

$f_x(x,y) = 1 \cdot e^{-3y} + \cos(2x-5y)(2+0)$
$= e^{-3y} + 2\cos(2x-5y)$

$f_y(x,y) = xe^{-3y}(-3) + \cos(2x-5y)(0-5)$
$= -3xe^{-3y} - 5\cos(2x-5y)$

$f_{xx}(x,y) = \overset{\frac{\partial}{\partial x}}{f_x}\left[e^{-3y} + 2\cos(2x-5y)\right] = 0 + 2\cdot(-\sin(2x-5y))2$
$= -4\sin(2x-5y)$

$f_{xy}(x,y) = \overset{\partial/\partial y}{f_y}\left[e^{-3y} + 2\cos(2x-5y)\right]$
$= e^{-3y}(-3) + 2(-\sin(2x-5y))(-5)$
$= -3e^{-3y} + 10\sin(2x-5y)$

$f_{yy}(x,y) = f_y\left[-3xe^{-3y} - 5\cos(2x-5y)\right]$
$= -3xe^{-3y}(-3) - 5(-\sin(2x-5y))(-5)$
$= 9xe^{-3y} - 25\sin(2x-5y)$

$f_{yx}(x,y) = f_x\left[-3xe^{-3y} - 5\cos(2x-5y)\right]$
$= -3e^{-3y} - 5(-\sin(2x-5y))(2)$
$= -3e^{-3y} + 10\sin(2x-5y)$

$\boxed{Ex}$ $f(x,y) = \dfrac{y}{2x+3y}$

Find $f_{yx}(x,y)$

$$f_y(x,y) = \frac{\overbrace{(2x+3y)\cdot 1 - y(3)}^{2x+3y-3y}}{(2x+3y)^2} = \frac{2x}{(2x+3y)^2}$$

$$f_{yx}(x,y) = f_x\left[\frac{2x}{(2x+3y)^2}\right]$$

$$= \frac{(2x+3y)^2\, 2 - 2x\, 2(2x+3y)^1\, 2}{(2x+3y)^4}$$

$$= \frac{(2x+3)\left[(2x+3y)\cdot 2 - 8x\right]}{(2x+3y)^4}$$

$$= \frac{-4x+6y}{(2x+3y)^3}$$

Find $f_{yy}(x,y) = f_y\left[\dfrac{2x}{(2x+3y)^2}\right] = f_y\left[2x(2x+3y)^{-2}\right]$

$$= 2x\cdot -2(2x+3y)^{-3}\cdot 3 = \frac{-12x}{(2x+3y)^3}$$